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Some Aspects of Modelling the Water Exchange in Kattegat

by

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Abstract

Two approaches to modelling water exchange and conservation are considered with the purpose of creating a basis for the development of ecological transport models for transient zones such as the Kattegat.

The first approach is a matrix model - a time-discrete Markov model of water exchange with a ½-lunar-day as the physical time-base. Primary data are the differences in average salinities between different positions and the situation is exemplified by considering the Kattegat-Baltic system as a simple, two-layer channel and using stationary box-exchange principles.

The second approach is a series of separate autoregressive models of water conservation exemplified by considering stationary, first-order time-discrete processes, AR(1). Each model relates to a given position (box) and the input data used are time-series of 10-days-average of salinities for the position in question.

A few results are given in order to compare the two approaches and to indicate their applicability but this paper is primarily to stimulate discussion.

1. INTRODUCTION.

The development of mathematical models of biomass production in the marine food-chain are of considerable interest for the fisheries, and for entrophication and pollution studies.

But, to be reliable and useful, such models must, as a minimum, include a description of the biological rules that govern species interaction and this, again, more or less implies that, a complete account of phosphorus (or, some other measure of biomass equivalents) is kept. The result of that ecosystem models of species interaction attain a large size even in cases in which the decriptions of chemical and physical processes are cut down to a minimum; see the Andersen and Ursin North Sea Model.
The remarks above indicate some of the reasons for initiating the present study. The need for developing appropriate, physical transport models to serve as a skeleton for the description of biological processes, in particular primary and secondary production, is recognised by most ecosystem modellers. But, on the other hand, such an integrated approach cannot be performed by operating with a large model of species interaction for each column of water in some selected sea grid, that is, if we want to avoid extremely costly, non-operational, supra-sized ecosystem models.
The solutions to these problems clearly lie in synthetized approaches to the essential interactions of physical, chemical and biological processes at various scales, and this, unfortunately, requires a level of interdisciplinary knowledge which the present authors do not possess.

But, before we forget about all the relevant biological and chemical processes for production in the sea, it may at least be noted that these processes represent very different time scales (or time constants).

And for this reason alone it seems useful to develop simple transport models that are able to say something about the positions in the sea at which it is likely to find a given mass of water after various periods of time.

This is what this paper is all about.. But there is another reason for restricting ourselves to simple models - the present authors are not physical oceanographers! All constructive criticism will be most wellcome.

2. TIME-DISCRETE MODEL PRINCIPLES APPLIED.

2.1 THE MATRIX APPROACH TO MODELLING EXCHANGE OF WATER.

The sea is partitioned into compartments or boxes. During a tidal period, the series of event are perceived as follows: (1) water is flowing into the individual box, (2) a mixing of the water masses in the box takes place when the tide is changing, and (3) water flows out of the box. Thus, the time-unit ,u, is half a lunar day (a M2 period), i.e.

$$u = 12.42 \text{ hours} = \frac{1}{705.7} \text{ years} \quad (1)$$

The individual box must be so large that the exchange volumes between adjoining boxes in a time-unit are small compared to the box volumes. On the other hand, the individual box should not be so large that the assumption of mixing within a time-unit is unreasonable.

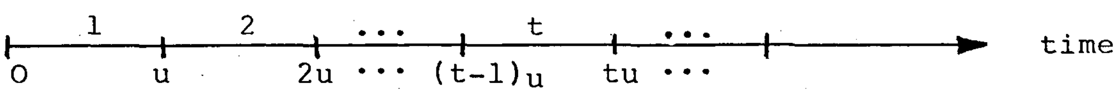


Fig 1 Time-notation in the matrix model. Time-period no.t starts at time (t-1)u and ends at time tu. The time-unit u is a "mixingtime".

In this time-discrete approach the continuity relations for box m read - in a first order approximation

$$S_t(m)\,\Delta V(m) + V_t(m)\,\Delta S(m) = \sum_{x \neq m} \left[S_t(x)\, Q_t(x,m) - S_t(m)\, Q_t(m,x) \right] \quad (2)$$

$$\Delta V(m) = \sum_{x \neq m} \left[Q_t(x,m) - Q_t(m,x) \right] \quad (3)$$

Here $Q_t(x,m)$ denotes the exchange volume (the transfer coefficient), i.e. the volume of water flowing from box x into box m during the time-period t. $S_t(m)$ and $V_t(m)$ are the (average) salinity and the volume of water of box m at the start of time-period t, respectively. The Δ designates the changes during the time-unit, i.e.

$$\Delta V(m) = V_{t+1}(m) - V_t(m)$$
$$\Delta S(m) = S_{t+1}(m) - S_t(m) \qquad (4)$$

Define

$$a(x,m) = \frac{Q(x,m)}{V(m)} \quad ; \; x \neq m \qquad (5)$$

$$a(m,m) = 1 - \sum_{x \neq m} a(x,m) \qquad (6)$$

Where the subscript t has been omitted for the sake of brevity. Dividing Eq.(2) with V(m), utilizing Eq.(3), and inserting the a-elements defined above, yield

$$S_{t+1}(m) = s_t(m)a_t(m,m) + \sum_{x \neq m} S_t(m)a_t(x,m) \qquad (7)$$

That is, in matrix notation,

$$\underline{S}_{t+1} = \underline{S}_t \underline{\underline{A}}_t \qquad (8)$$

Where $\underline{S}$ is a row-vector giving the box salinities and $\underline{\underline{A}}$ is a square, exchange matrix having a(x,m) as element in the x'th row and the m'th column.

The exchange matrix may be interpreted stochastically because all its elements are non-negative and the column-sums are one. Let us consider the m'th column which represents box m. The diagonal element, $a_t(m,m)$, gives the volume-fraction of box m (at the start of time-period t) that remains in the box (i.e. conserved) during time-period no.t.

The off-diagonal element $A_t(x,m)$ gives the volume-fraction of box m that is exchanged with water from box x during time-period No. t. Thus, the off-diagonal elements in column m denote the origin of the water that has been exchanged in box m during a time-unit. In a stochastic interpretation we may say that a water molecule selected at random from box m (in time-period t+1) originates from one of the boxes (in time-period t) with probabilities given by the m'th column of the exchange matrix, $\underline{\underline{A}}_t$.

In the example considered in this paper interest is focused on the simple stationary case. That is, the box salinities are considered constant from one time-period to the next:

$$S_{t+1}(m) = S_t(m) = S(m) \tag{9}$$

and we then want to determine the corresponding stationary exchange-pattern:

$$Q_t(m,x) = Q(m,x) \tag{10}$$

In other words, we must solve Eqs. (2) and (3) which now take the simple form

$$\sum_{x \neq m} S(x)Q(x,m) = S(m)\sum_{x \neq m} Q(m,x) = S(m)\sum_{x \neq m} Q(x,m) \tag{11}$$

The equations express that the net supply of salt and water to box m is zero during a time-period. A unique solution, of course, only exists if the number of unknown Q's is in balance with the number of equations.

With a complete set of Q and V values, the constant exchange matrix, $\underline{\underline{A}}$, is determined from Eqs. (5) and (6). Thus, according to Eq. (8), the exchange of water as time elapses is simply computed as $\underline{\underline{A}}, \underline{\underline{A}}^2, \underline{\underline{A}}^3, \cdots, \underline{\underline{A}}^t$. That is, the exchange-of-water-situation is completely determined by $\underline{\underline{A}}$ and the underlying time-unit u. The columnsums in $\underline{\underline{A}}^t$ are one and the individual elements of the m'th column give the origin (t time-units back in time) of the water-make-up of box m at time tu. After a long time (i.e. $t \to \infty$) the original box m water is completely replaced by the permanent sources, i.e. freshwater and oceanwater (represented as boxes with infinite volume). The elapse time required to reach this equilibrium situation with a given accuracy is determined by the eigenvalues of $\underline{\underline{A}}$.

2.2 The autoregressive approach

The primary data input to the stationary matrix exchange model is the differences in average salinities between different boxes. Thus, only the information on the average salinity for a certain period of time is utilised for each box in question. The autoregressive approach is independent of the matrix approach in the sense that only information on changes with time in the salinities at a fixed position (box) is utilised.

To illustrate the principles, we consider the simplest possible model - a stationary, first order, time-discrete autoregressive process:

$$S_t(m) = P(m) + C(m)S_{t-1}(m) + Z_t(m) \qquad (12)$$

As in the previous sextion, t denotes a time-period which, however, is not necessarily of duration half a lunar day.
The model simply states that the average salinity in box m in period t is a linear function of the average salinity in the box for the previous period of time plus a random deviation, $Z_t(m)$. The process is assumed to be stationary, i.e. the Z_t's, t=1,2,..., are assumed to be stochastically independent and identical distributed variables with mean zero.

In the example consider in this paper, the autoregressive parameter C(m) is simply estimated from the salinity time-series by the autocovariance of first order (i.e. the Yule-Walker estimate).

In the present context, C(M) is interpreted as the fraction of water in the box that is conserved (i.e. not exchanged) from one period of time to the next.

3. AN EXAMPLE

The box partitioning of the Kattegat-Baltic fiord system is depicted in Fig. 2 and input data are given in Table 1. Box volumes for Kattegat are calculated by U. Ehlin from the SMHI special data base. Salinities are from K.P.Andersen's data analysis of the Danish Belt Project. Inflow of rivers are taken from Falkenberg and Mikulski (1974) and Svansson (1975).

The box system comprises 30 unknown Q's but only 20 equations (i.e. 10 finite boxes) are available. It is therefore necessary to introduce 10 Q-constraints. We assume that the inflow of ocean water primary takes place in the bottom channel and similar that the outflow of the brackish Baltic water occurs primarily in upper channel. The reversed exchange pattern is then assumed to equal fixed fractions ($0<K<1$) of the related Q's in the primary flow pattern. As an example

$$Q(10,8) = K(10,8)Q(8,10)$$

The 10 K's are divided into four groups - the K's being equal within groups - see Table 3. One group comprises K(3,5) and K(5,7) and these are put to zero always. The value of the K's in the three other groups are denoted by a,b.and c, respectively.

Autoregressive parameters are estimated based on the 1972 time-series of 10-days-mean-salinities (Danish Belt Project), averaging over daily measurements at 0,5 and 10 m depths. The light vess at Laesoe N. and Aalborg Bay are ancored in the box 10 area whereas Anholt N. and Kattegat SW are located in the box 8 area.

4. RESULTS AND DISCUSSION

If not stated otherwise, the Q-constants are a=0.3, b=0.2 and c=0.1.

In Table 2 water transport on a yearly basis - computed from the matrix model - is shown together with the results from the Aage J.C. Jensen model - see Fig. 3. Here a=b=c=0 because Jensen'S model is a one way running model. The agreement in the results is not convincing.

Table 3 gives water conservation in percent on a 128 time-period basis (66.24 days), i.e. the diagonal elements $a_{128}(m,m)$, m= 2,3,...,11 of the matrix $\underline{\underline{A}}^{128}$. The case a=b=0.7 and c=0 produced a negative Q value. It appears that water conservation, apart from box 5 and box 11, is not affected very much by the choice of Q-constrain values.

Approximately 1% of the large Baltic boxes (2 and 3) are exchanged on a 2 months basis. Boxes 4 to 7 serve as buffers between the Baltic and Kattegat. In Kattegat, the water exchange in the upper boxes takes place much faster than in the bottom boxes(i.e. below the halocline which is set to 10 m). 9% of the initial water in box 8 is still present after 2 months. In box 10 the figure is only about 3%. Fig. 4 gives a better picture of the decline in water conservation as time elapses. The results are in reasonable good accordance with the autoregressive model - the straight lines on the Figure.

The off-diagonal elements in the matrix provides information on the origin of the water masses. Fig. 5 shows that the contents of box 8 water in box 10 increases to a maximum of 22% after 17.5 days. Below the halocline, the inflow of North Sea - Skagerak water results also in 22% box 11 water in box 9, but this maximum is first reached after 60 days. Considerations of this type may be useful in relation to the consequences of sudden events such as outslip disasters or extreme plankton blooms.

After 1.5 years all the Kattegat boxes contains less than 1% of other sources than the permanent sources - including Baltic waters. The content of Baltic water in the Kattegat boxes reaches its maximum after 1 to 12 years after which it declines and is less than 1% - in all boxes - after 190 years.

Returning to water transport on a yearly baiss, Table 4 shows the water exchange through the 5 vertical sections - see Fig. 2 and the map in Table 1 for reference - as outflow from/ inflow to the Baltic, and the resluting net outflow. The fact that the net outflow is independent of the choice of constants (i.e. a,b and c) only reflects that the model reproduces the constant freshwater input with reasonable accuracy. As regards the inflow and outflow, it seems obvious that the effect of increasing constants is that "the mill is grinding faster". Soskin finds the outflow through the Danish straits to 1188 km^3/y and the inflow to 1660 km^3/y. We are not quite sure but find it most reasonable to relate these figures to section III.

Very few data on mixing processes in Kattegat are available to calibrate the matrix model. The only real conservative tracer - salt - has already been used as input. However, the bottom temperature is a fairly conservative parameter. We focus on box 9. The temperatures in the surrounding boxes (7,8 and 11) are approximated by second order Fourier expressions:

$$T_d = \overline{T} + k_1\cos(dP) + k_2\sin(dP) + k_3\cos(2dP) + k_4\sin(2dP) \quad (13)$$

T_d is the temperature on day No d of the calender year and $\overline{T}$ the annual mean temperature. P is a constant, $2\pi/365.24$. The k's were estimated from monthly mean temperature data from Laesoe N (box 11), Anholt N (box 8 and 9) and Kattegat SW (box 7). We then assume that the temperature - starting at day 45 - in boxes 7,8 and 11 are given by these estimated models. The temperature in box 9 starts on day 45 at the Anholt-N-estimated-value. But from day 45, onwards, the box 9 temperature is computed in time-steps of half a lunar day using the matrix model with the temperature of box 7,8 and 11 as input, i.e. the temperature in box 9 is computed by "mixing box 7,8 and 11 temperatures" according to the exchange matrix.

The results of these calculations are plotted in Fig. 6. The agreement with the estimated Anholt N model, Eq.(13), is fairly good apart from the descending part of the curves. Perhaps the latter disagreement is due to the fact that the transport of heat from the deep layers to the surface depends less on the mixing processes than the transport the opposite way. At any rate, considering that monthly means are not suited for estimating the maximum and that Anholt N is ancored in the northeast-corner of box 9, we should not go too far in the interpretation of Fig. 6.

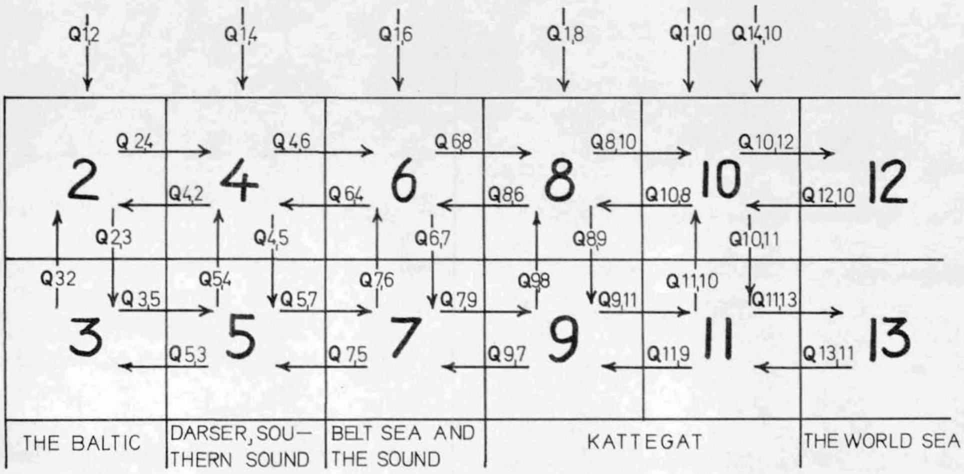


Fig.2. The box model

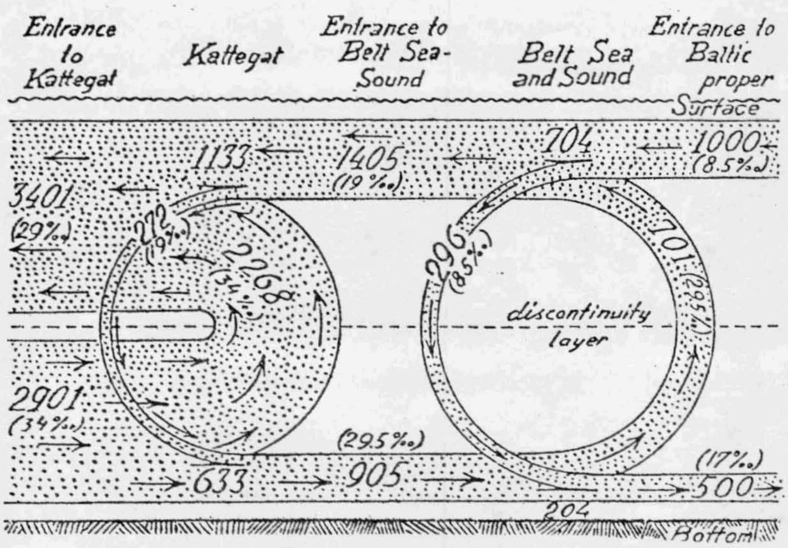


Fig.3. Aage J. C. Jensens model (1940)

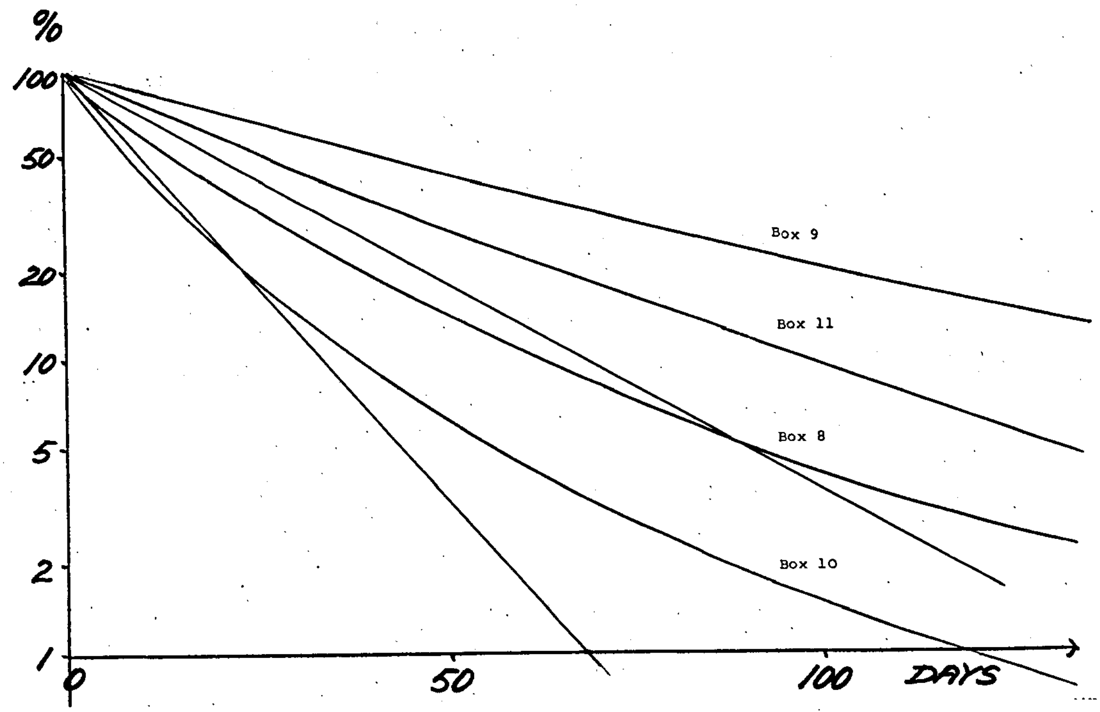


Fig.4. Water conservation in Kattegat boxes estimated by the box model and by autoregressive analysis on observations from the light vessels Ålborg Bugt, Laesoe N, Anholt N and Kattegat SW.

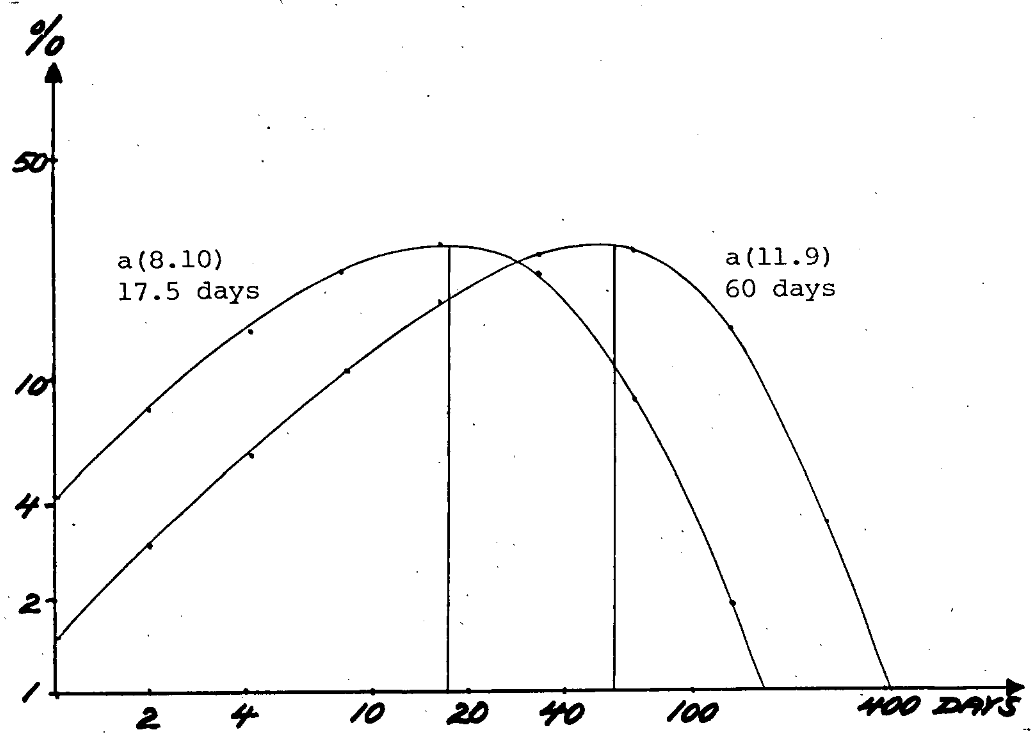
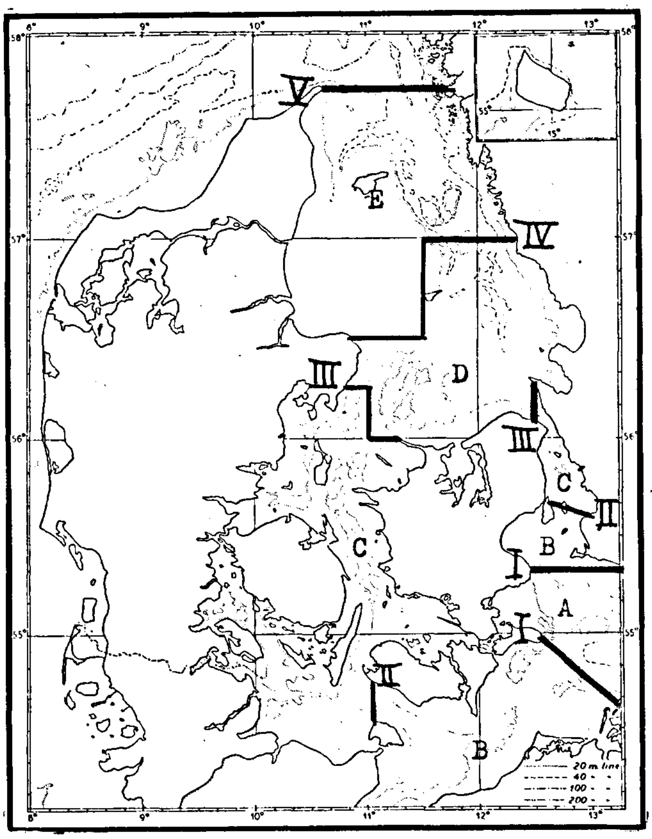


Fig.5. Content of box 8 water in box 10 and of box 11 water in box 9.

Table 1. Input data to the box model



Inflow of river water

		km^3/y
A	Q(1,2)	471.4
B	Q(1,4)	4.0
C	Q(1,6)	2.0
D	Q(1,8)	5.0
E	Q(1,10)	19.1

Inflow of Sea Water from the Limfjord Q(14,10) approx. 4 km^3/y

Salinities

A	S(2) = 7.50‰	S(3) = 11.50‰
B	S(4) =12.13‰	S(5) = 18.20‰
C	S(6) =18.10‰	S(7) = 29.06‰
D	S(8) =21.56‰	S(9) = 32.28‰
E	S(10)=26.16‰	S(11)= 33.66‰
	S(12)=33.10‰	S(13)= 34.75‰
Limfjord.........		S(14)= 25.00‰

Box Volumes

A	V(2) = 15.000.0 km^3	V(3) = 6.200.0 km^3
B	V(4) = 94.0 km^3	V(5) = 25.0 km^3
C	V(6) = 92.0 km^3	V(7) = 146.0 km^3
D	V(8) = 100.3 km^3	V(9) = 135.9 km^3
E	V(10) = 105.3 km^3	V(11)= 141.3 km^3

Step time

$$u = 12.42 \text{ hours} = \frac{1}{705.7} \text{ years}$$

Table 2.

Water transport pr. year.in km^3

Aage J.C.Jensen	14 box model	
1 000	802	Q(2,4)
600	331	Q(5,3)
701	1 068	Q(5,4)+Q(7,6)
296	789	Q(4,5)+Q(6,7)
1 405	1 087	Q(6,8)
905	610	Q(9,7)
2 268	1 176	Q(11,10)+Q(9,8)
272	317	Q(10,11)+Q(8,9)
3 401	1 974	Q(10,12)
2 901	1 468	Q(13,11)

a,b,c=0

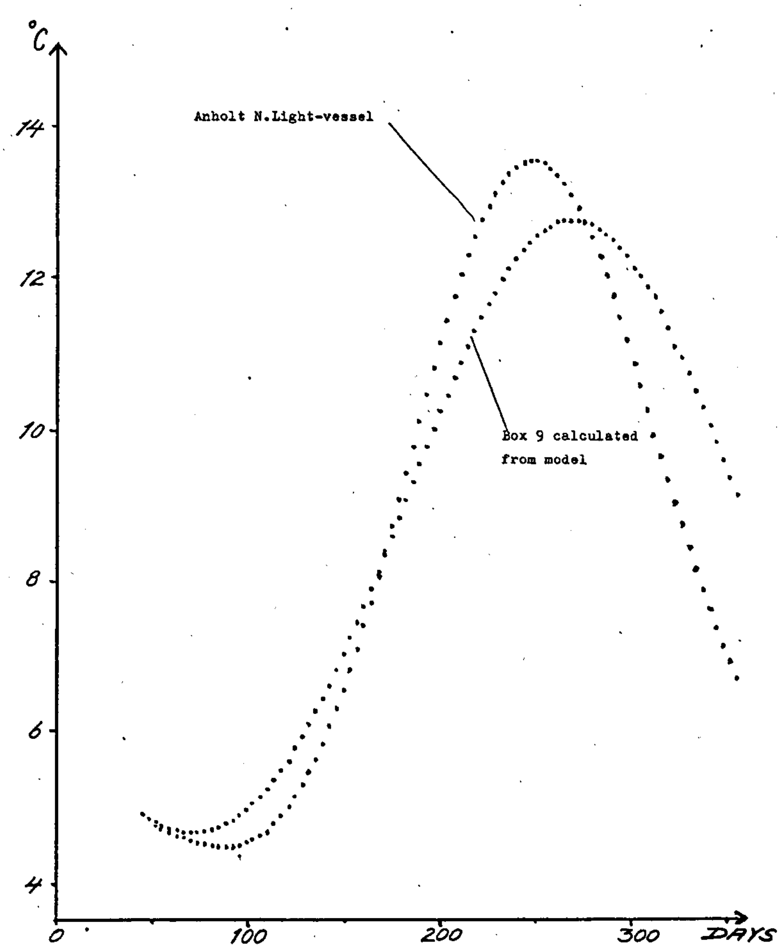


Fig.6. Temperatures near the bottom at light vessel Anholt N, compared with temperatures calculated from the box model

Table 3.

Water Conversation o/o after 128 periods

constants			box no									
a	b	c	2	3	4	5	6	7	8	9	10	11
0	0	0	98.4	97.5	15.o	3.10	10.0	40.4	7.94	28.5	2.97	12.1
0.2	0.2	0	98.5	98.1	12.1	3.24	10.1	43.9	8.74	33.4	3.28	18.5
0.3	0.3	0	98.5	98.5	10.8	3.72	10.1	46.4	8.97	37.0	3.39	23.6
o.4	0.4	0	98.7	98.8	9.53	5.17	9.99	49.7	9.o9	41.7	3.48	30.6
0.5	0.5	0	98.8	98.8	8.26	10.2	9.79	54.3	9.10	48.0	3.56	40.5
0.6	0.6	0	98.9	99.8	6.91	32.4	9.49	61.0	9.01	57.2	3.64	54.5
0.7	0.7	0	-	-	-	-	-	-	-	-	-	-
0.2	0.2	0.1	98.5	98.1	12.2	3.25	10.2	42.0	8.87	30.6	3.30	15.4
0.3	0.2	0.1	98.5	98.1	12.2	3.25	10.1	42.0	9.04	33.8	3.49	19.9
0.5	0.5	0.2	98.8	99.3	8.27	10.2	9.84	49.9	9.24	40.9	3.63	3o.9
0.6	0.5	0.3	98.8	99.3	8.27	10.2	9.82	47.6	9.48	45.2	3.90	39.0

$a = K_{10,8}\ \ K_{12,10}$

$b = K_{4,2}\ \ K_{6,4}\ \ K_{8,6}$

$c = K_{7,9}\ \ K_{9,11}\ \ K_{11,13}$

$O = K_{3,5}\ \ K_{5,7}$

Table 4.

Water Exchange through the Sections

Section	Outflow	Inflow	net. Outflow	
I	806.9 km^3/y	332.8 km^3/y	472.0 km^3/y	a=0
II	816.9 "	342.9 "	475.0 "	b=0
III	1089.3 "	605.1 "	477.1 "	c=0
IV	1341.4 "	857.3 "	482.1 "	
V	1976.8 "	1472.5 "	505.3 "	
I	904.6 "	433.5 "	471.6 "	a=0.3
II	937.7 "	462.2 "	475.6 "	b=0.2
III	1323.8 "	847.2 "	477.5 "	c=0.1
IV	1712.1 "	1230.0 "	482.5 "	
V	2253.7 "	1748.5 "	505.7 "	
I	1119.7 "	648.6 "	471.6 "	a=0.6
II	1206.8 "	731.4 "	475.6 "	b=0.5
III	1847.7 "	1370.1 "	477.5 "	c=0.3
IV	2263.6 "	1781.5 "	482.5 "	
V	2446.7 "	1941.5 "	505.7 "	

References

Det danske Hedeselskabs kulturtekniske afdelings hydrometriske undersøgelser 1955-60 (1968).

Andersen, K.P. and Ursin, E.: A multispecies extension to the Beverton and Holt Theory of Fishing with accounts of phosphorus circulation and primary production. Medd. Danm. Fiskeri-og Havunders.,N.S. p.p. 319-435 (1977).

Ehlin, U. et al.: Computerbased Calculations of Volumes of the Baltic Areas. Proc.Conf.IX.Baltic Oceanographers.p.114. (1974).

Falkenberg, M. and Mikulski, Z.: Hydrology of the Baltic Sea. Stockholm and Warszawa (1974).

Jensen, Aage J.C.: The Influence of the Currents in the Danish Waters on the Surface Temperature in the Winter, and of the Winter Temperature of the Air. Medd.Komm.f.Danm.Fisk. og Hav. Ser. Hydrografi III. (1940).

Soskin, I.M.: Fleråriga Förandringar av Östersjöens hydrologiska Karakteristika (Swedish translation)(1963).

Svansson, A.: Physical and Chemical Oceanography of the Skagerac and the Kattegat. Fish Board Swe. (1975).